Liquid wires

Coiling rigid microfibers inside liquid droplets

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Liquid wires?

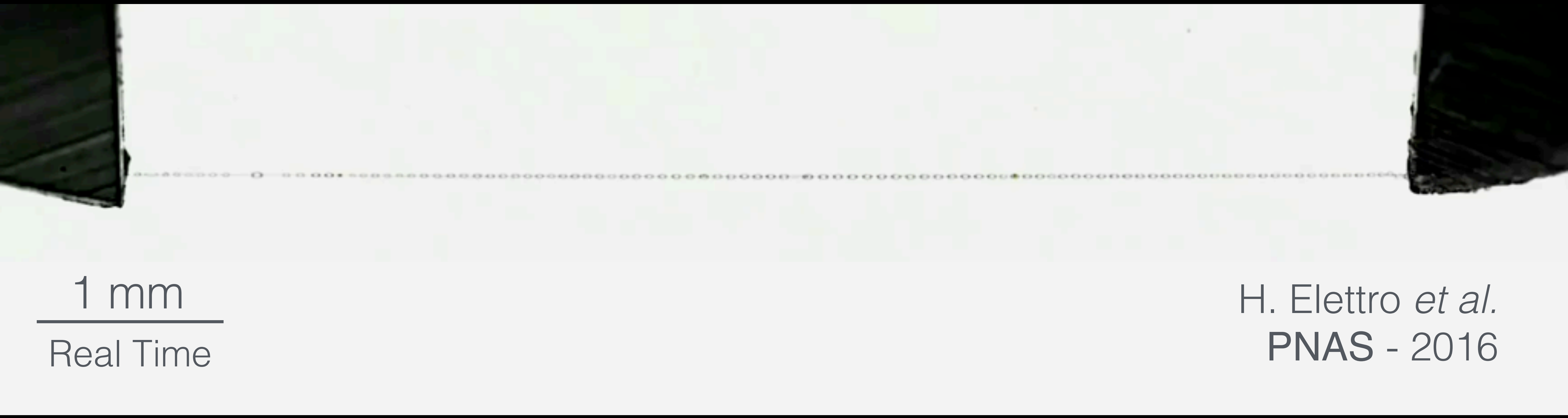
Biological inspiration

Nephila
golden orb weaver

Interesting properties
of its capture silk

Nephila capture silk

Its highly compressible property

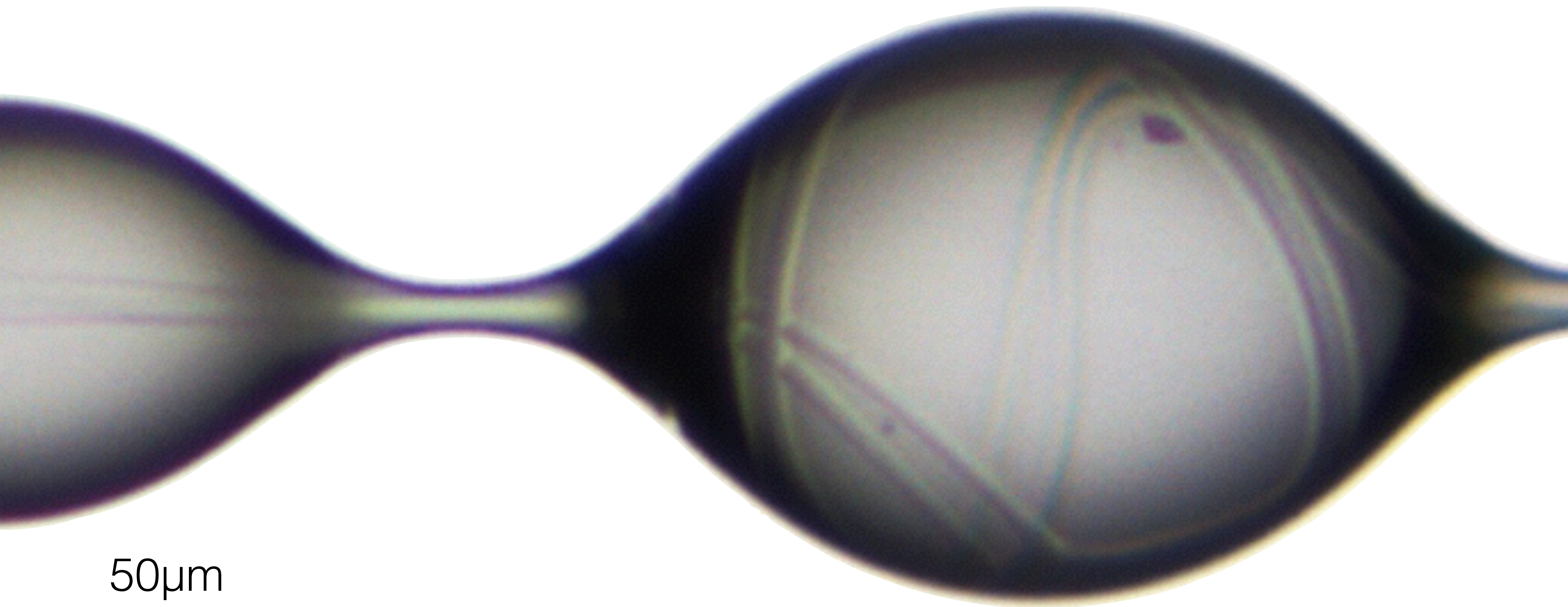


Nephila spider capture silk **thread** coated with small water **droplets**.
Throughout the **compression**, the fiber **does not sag**,
it remains under **tension**.

Video credit: Hervé Elettro

Nephila capture silk

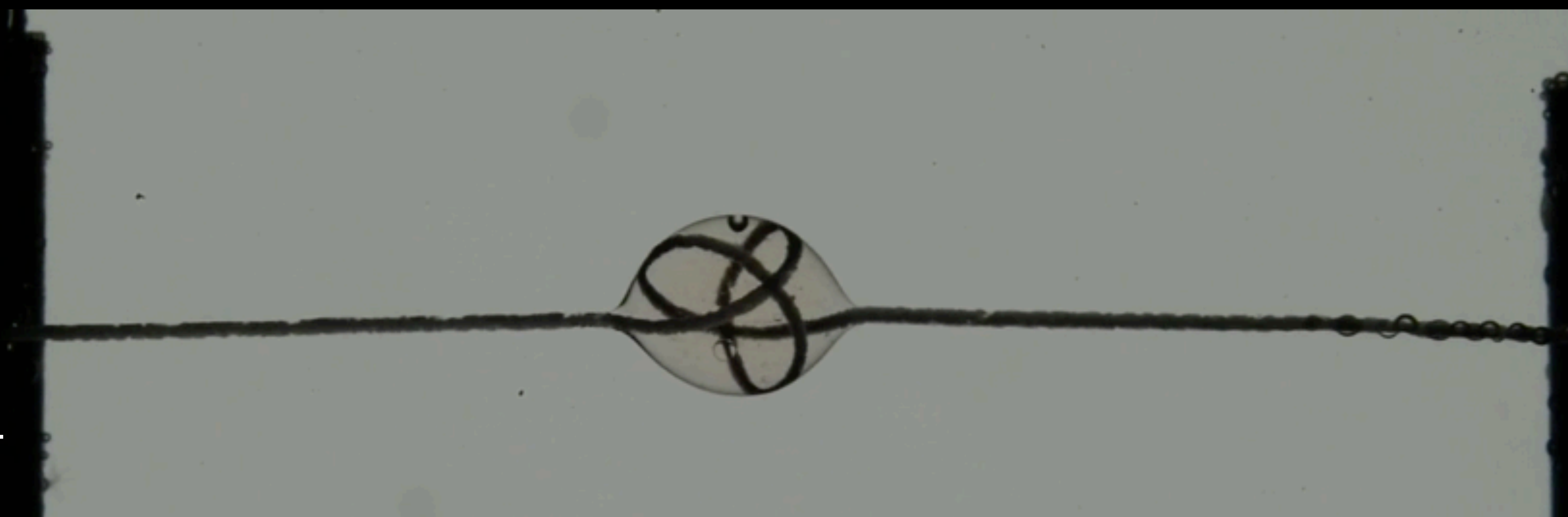
A closer look inside the droplets



During the **compression**,
the thread **spools** inside the water **droplets**.

Elastocapillary in-drop spooling

The movie



3 mm

Real Time

Silicone oil **droplet** on a RTV (silicone polymer) fiber.

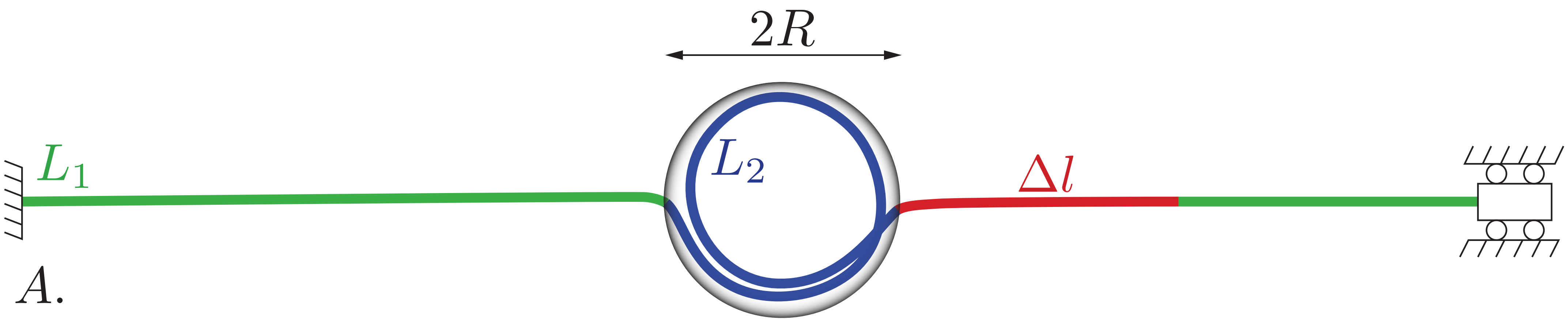
The system is **immersed** in a water bath.

Fiber: radius $a \approx 35$ μm, Young's modulus $E \approx 1$ Mpa.

Droplet : radius $R = 1.5$ mm, $\Delta\gamma \approx 40$ mN/m.

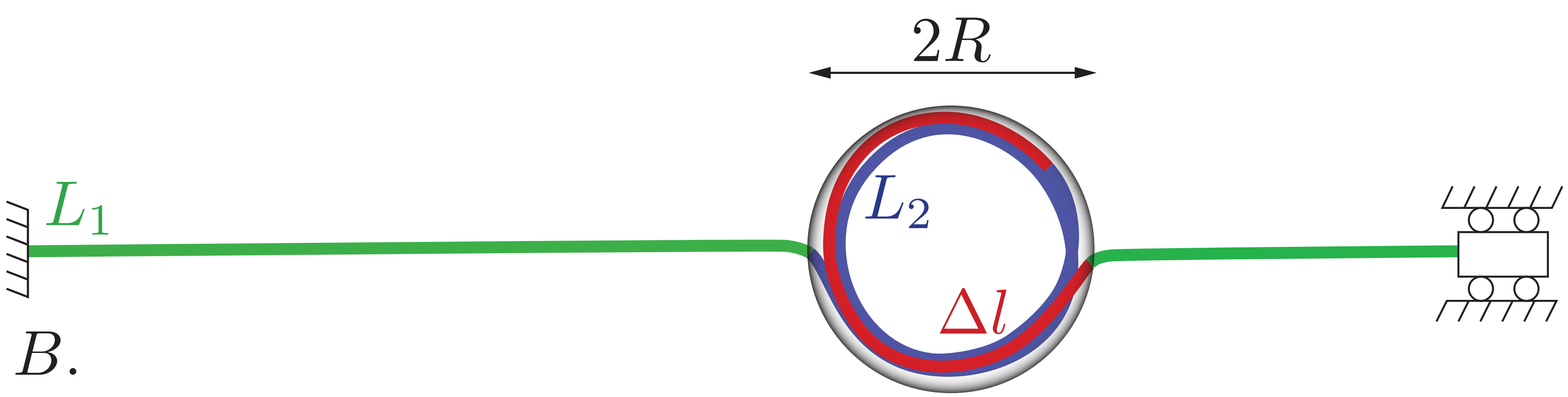
Elastocapillary in-drop spooling

The energetic approach



Solid-vapor interface energy | Solid-liquid interface energy | Bending energy | Solid-vapor interface energy

$$V_A = PL_1\gamma_{\mathrm{sv}} + PL_2\gamma_{\mathrm{sl}} + L_2\frac{1}{2}\frac{EI}{R^2} + P\Delta l\gamma_{\mathrm{sv}}$$



$$V_B = PL_1\gamma_{\mathrm{sv}} + PL_2\gamma_{\mathrm{sl}} + L_2\frac{1}{2}\frac{EI}{R^2} + P\Delta l\gamma_{\mathrm{sl}} + \Delta l\frac{1}{2}\frac{EI}{R^2}$$

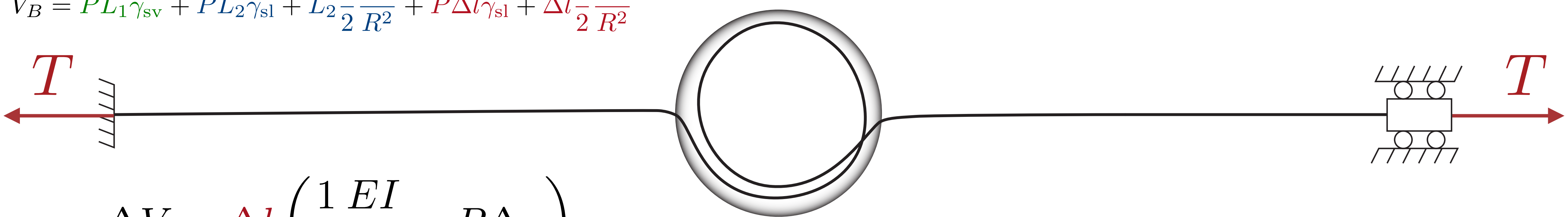
Solid-liquid interface energy | Bending energy

Elastocapillary in-drop spooling

The energetic approach

$$V_A = PL_1\gamma_{\rm sv} + PL_2\gamma_{\rm sl} + L_2\frac{1}{2}\frac{EI}{R^2} + P\Delta l\gamma_{\rm sv}$$

$$V_B = PL_1\gamma_{\rm sv} + PL_2\gamma_{\rm sl} + L_2\frac{1}{2}\frac{EI}{R^2} + P\Delta l\gamma_{\rm sl} + \Delta l\frac{1}{2}\frac{EI}{R^2}$$



$$\Delta V = \Delta l\left(\frac{1}{2}\frac{EI}{R^2} - P\Delta\gamma\right)$$

$$T = -\frac{\Delta V}{\Delta l} = \left(P\Delta\gamma - \frac{1}{2}\frac{EI}{R^2}\right)$$

$$T > 0\,?$$

$$2\pi a\gamma - \frac{\pi a^4 E}{8R^2} > 0$$

$$a < \left(\frac{16\gamma}{E}\right)^{1/3} R^{2/3} \;\sim 10\,\mu\mathrm{m}$$

Circular cross section fiber of radius a:

$$P = 2\pi a$$

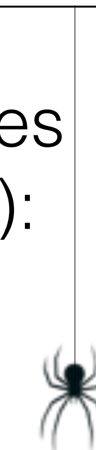
$$I = \frac{\pi a^4}{4}$$

Typical values (spider silk):

$R \sim 100\ \mu\mathrm{m}$

$E \sim 10\ \mathrm{MPa}$

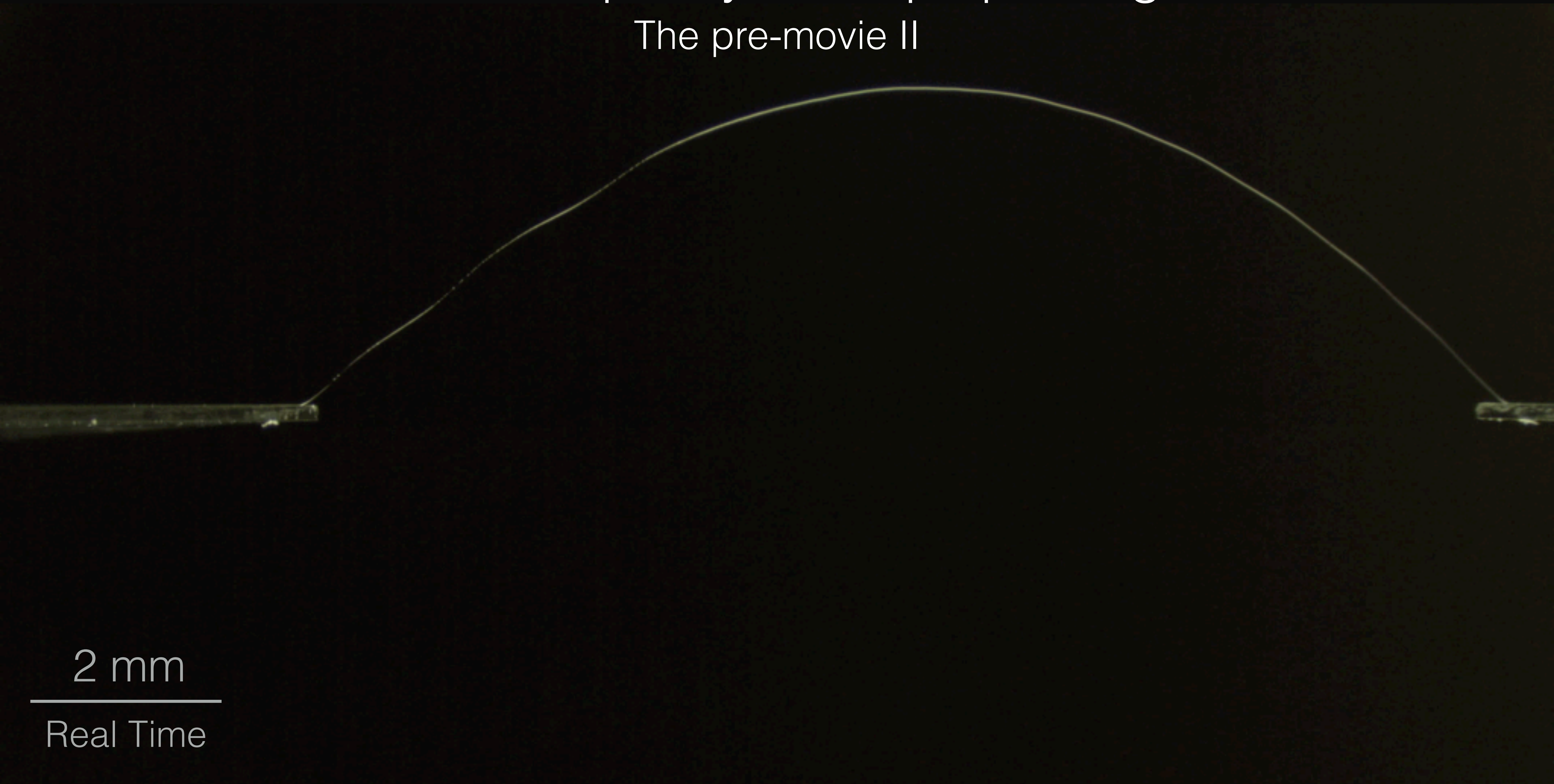
$\Delta\gamma \sim 50\ \mathrm{mN/m}$



If it is **thin** and **elastic** enough, a thread can coil inside a droplet!

Elastocapillary in-drop spooling

The pre-movie II



Bear thermoplastic polyurethane microfiber with **no** droplet sitting on it.

Fiber: radius a=3.3 μm, Young's modulus E=20 Mpa.

Elastocapillary in-drop spooling

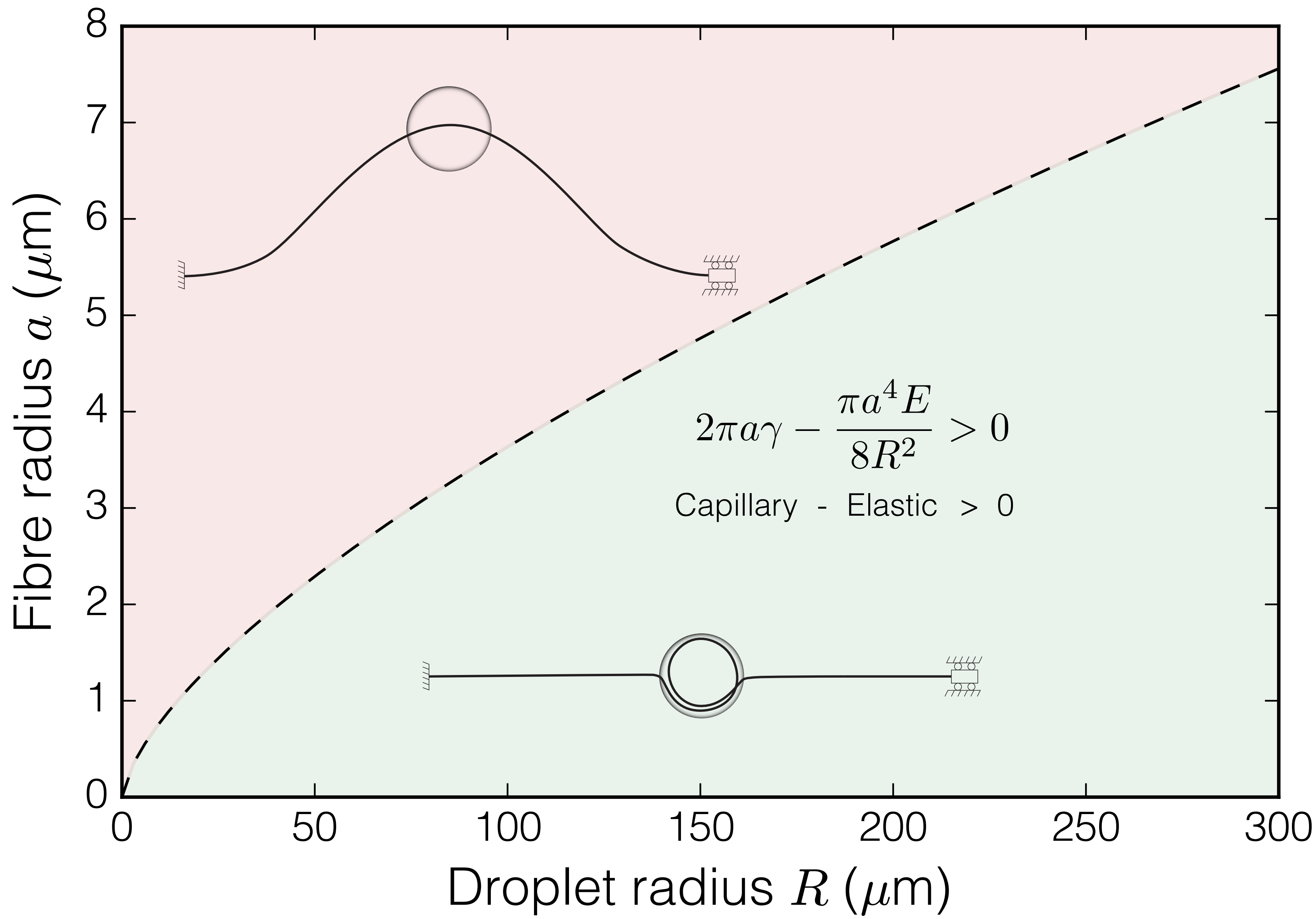
The movie II

Silicone oil **droplets** on said thermoplastic polyurethane **microfiber**.
An artificial ultra **compressible/extensible** device.

Fiber: radius a=3.3 µm, Young's modulus E=20 Mpa.
Final droplet : radius R=106 µm, γ=21 mN/m.

The coiling fibers

Phase diagram



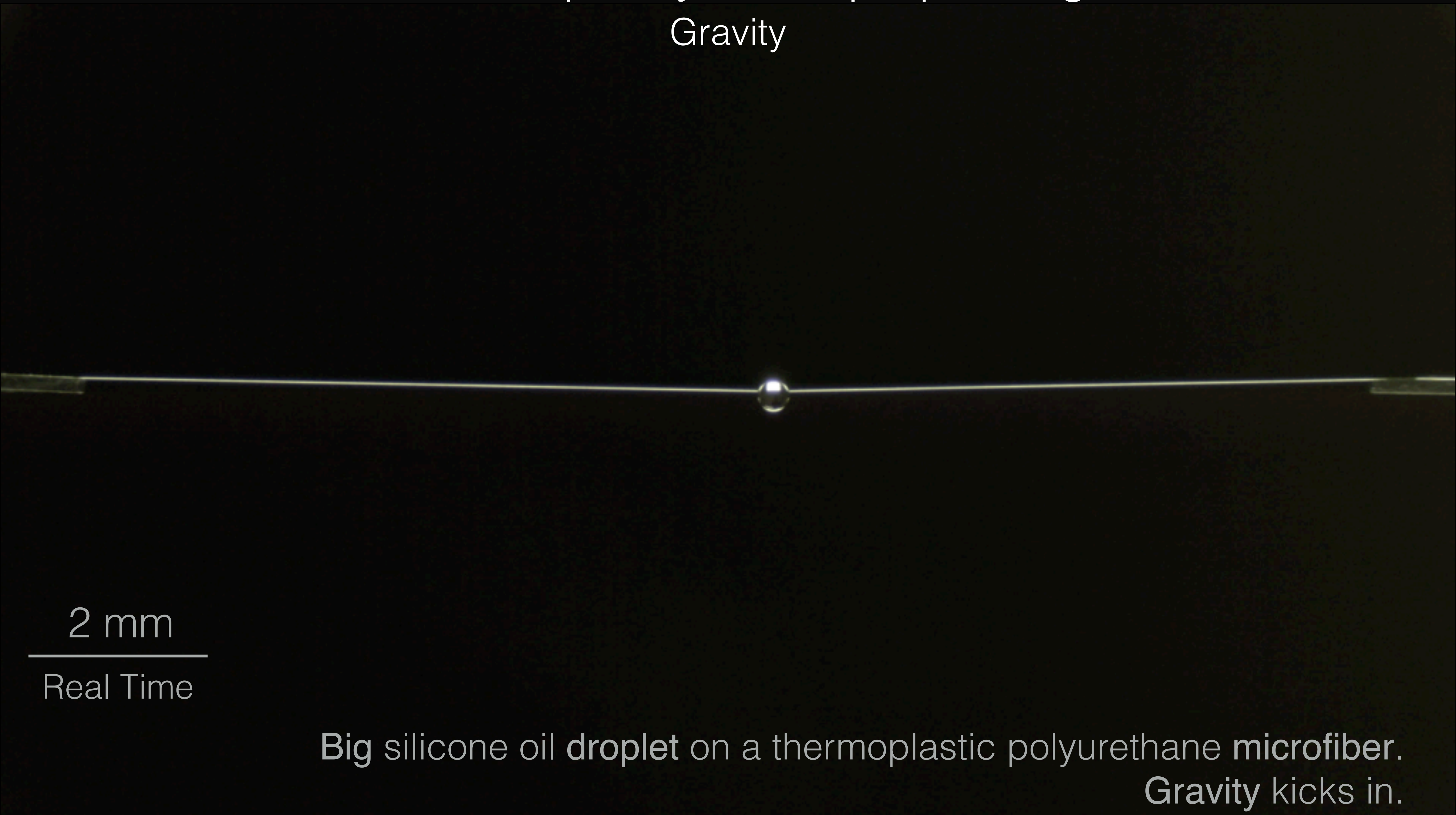
Phase diagram of **coiling** and **non-coiling** fibers

Fiber: Considered Young's modulus E = 70 MPa.

Droplet : $\gamma \approx 21$ mN/m.

Elastocapillary in-drop spooling

Gravity



Big silicone oil **droplet** on a thermoplastic polyurethane **microfiber**.

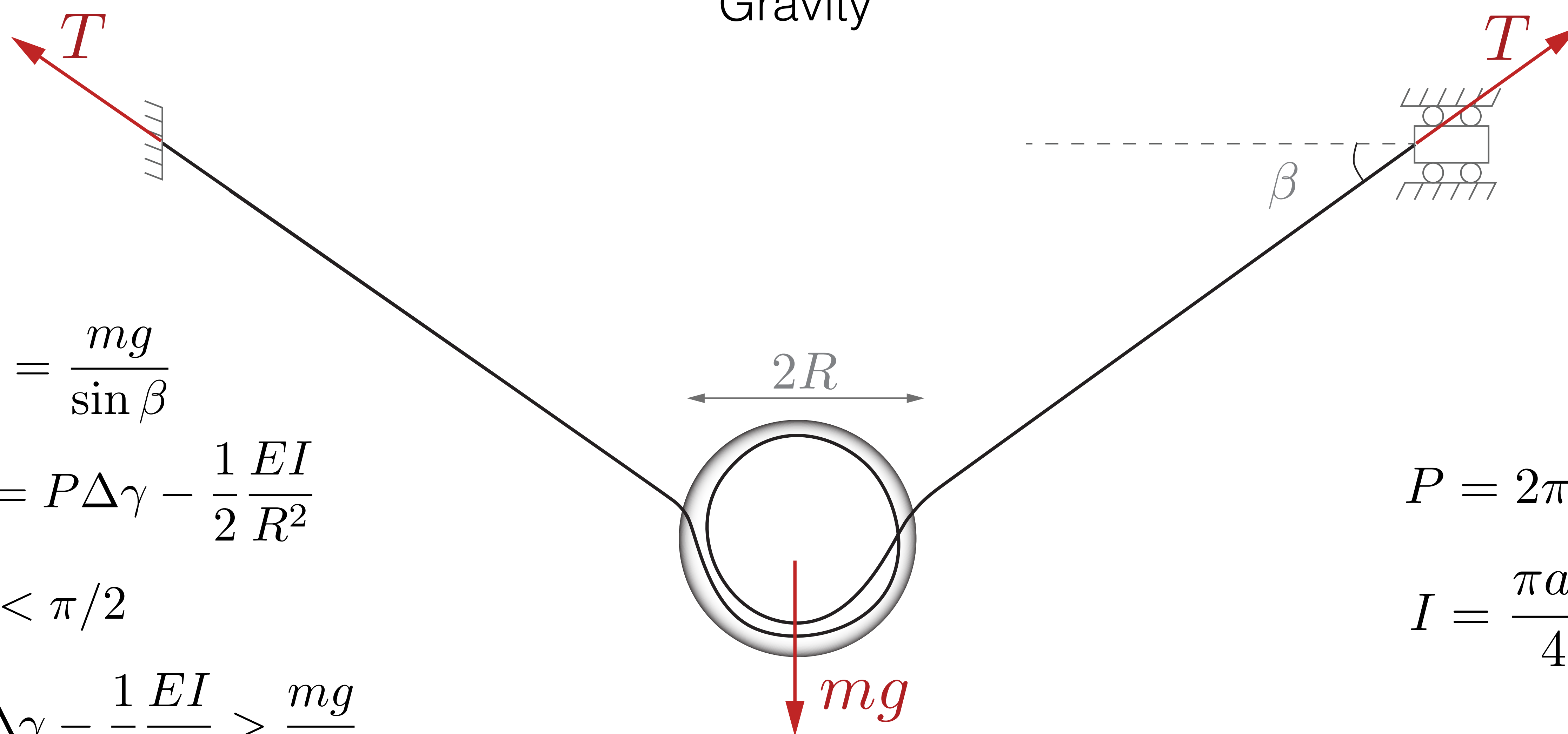
Gravity kicks in.

Fiber: radius a=3.4 µm, Young's modulus E=20 Mpa.

Droplet : radius R=430 µm, γ=20 mN/m.

Elastocapillary in-drop spooling

Gravity



$$2T = \frac{mg}{\sin\beta}$$

$$T = P\Delta\gamma - \frac{1}{2}\frac{EI}{R^2}$$

$$\beta < \pi/2$$

$$P\Delta\gamma - \frac{1}{2}\frac{EI}{R^2} > \frac{mg}{2}$$

$$P = 2\pi a$$

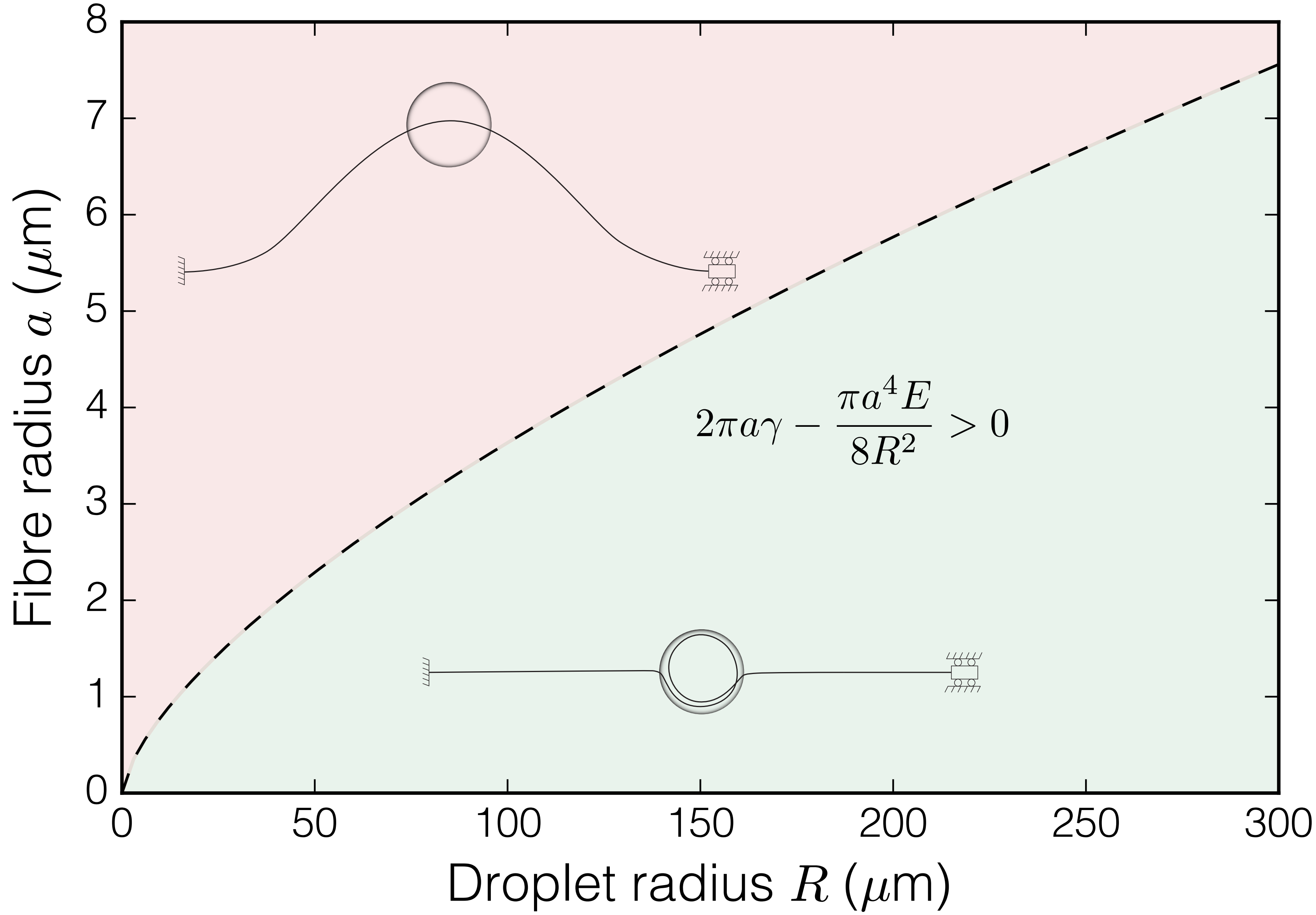
$$I = \frac{\pi a^4}{4}$$

$$2\pi a\gamma - \frac{\pi a^4 E}{8R^2} > \frac{4}{3}\frac{\pi\rho g R^3}{2}$$

$$2\pi a\gamma - \frac{\pi a^4 E}{8R^2} > \frac{4}{3}\frac{\pi\rho g R^3}{2}$$

The coilable fibers

Phase diagram



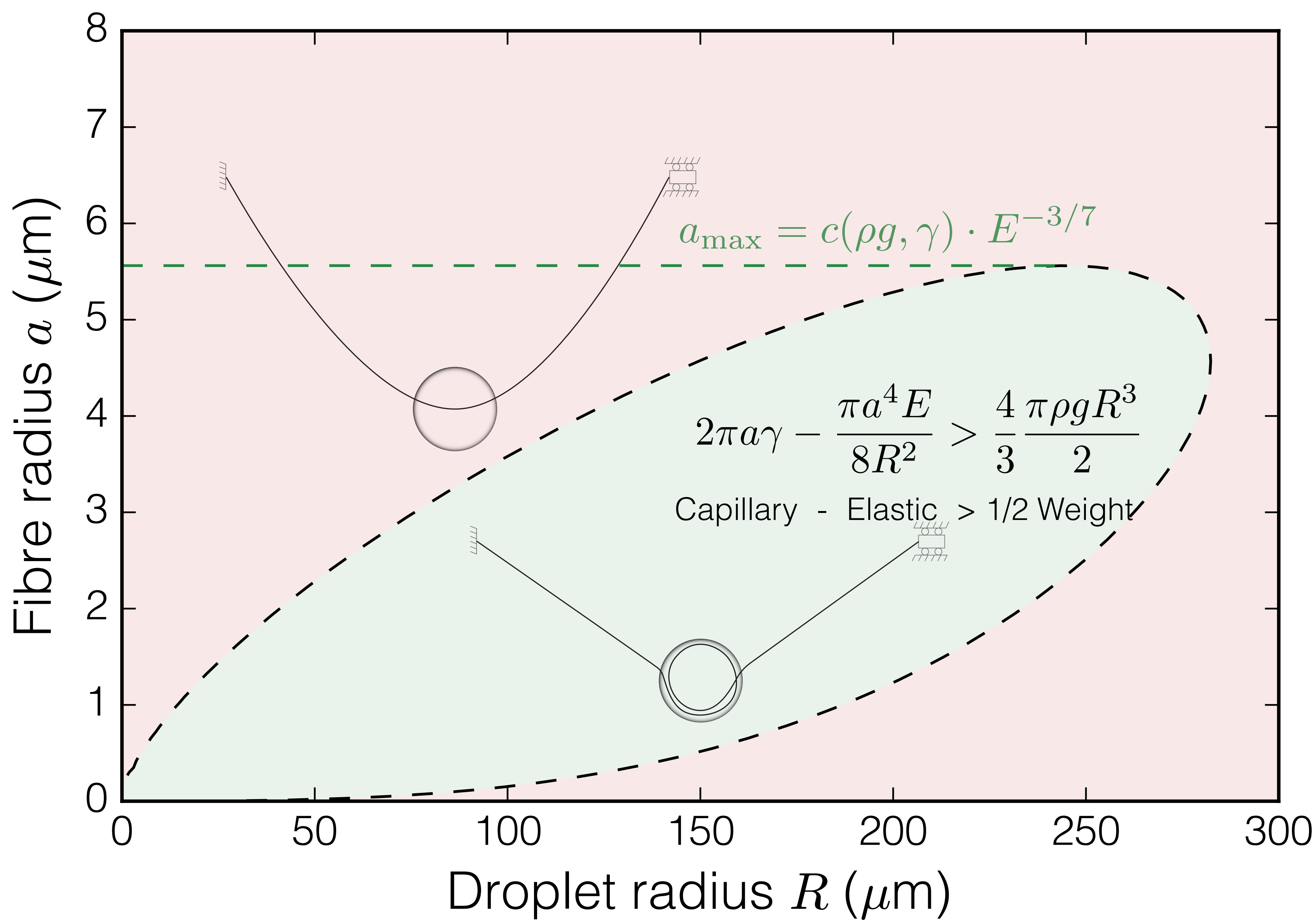
Phase diagram of **coilable** and **non-coilable** fibers

Fiber: Considered Young's modulus E = 70 MPa.

Droplet : γ=21 mN/m.

The coiling fibers

Phase diagram



Phase diagram of **coiling** and **non-coiling** fibers with **gravity**.

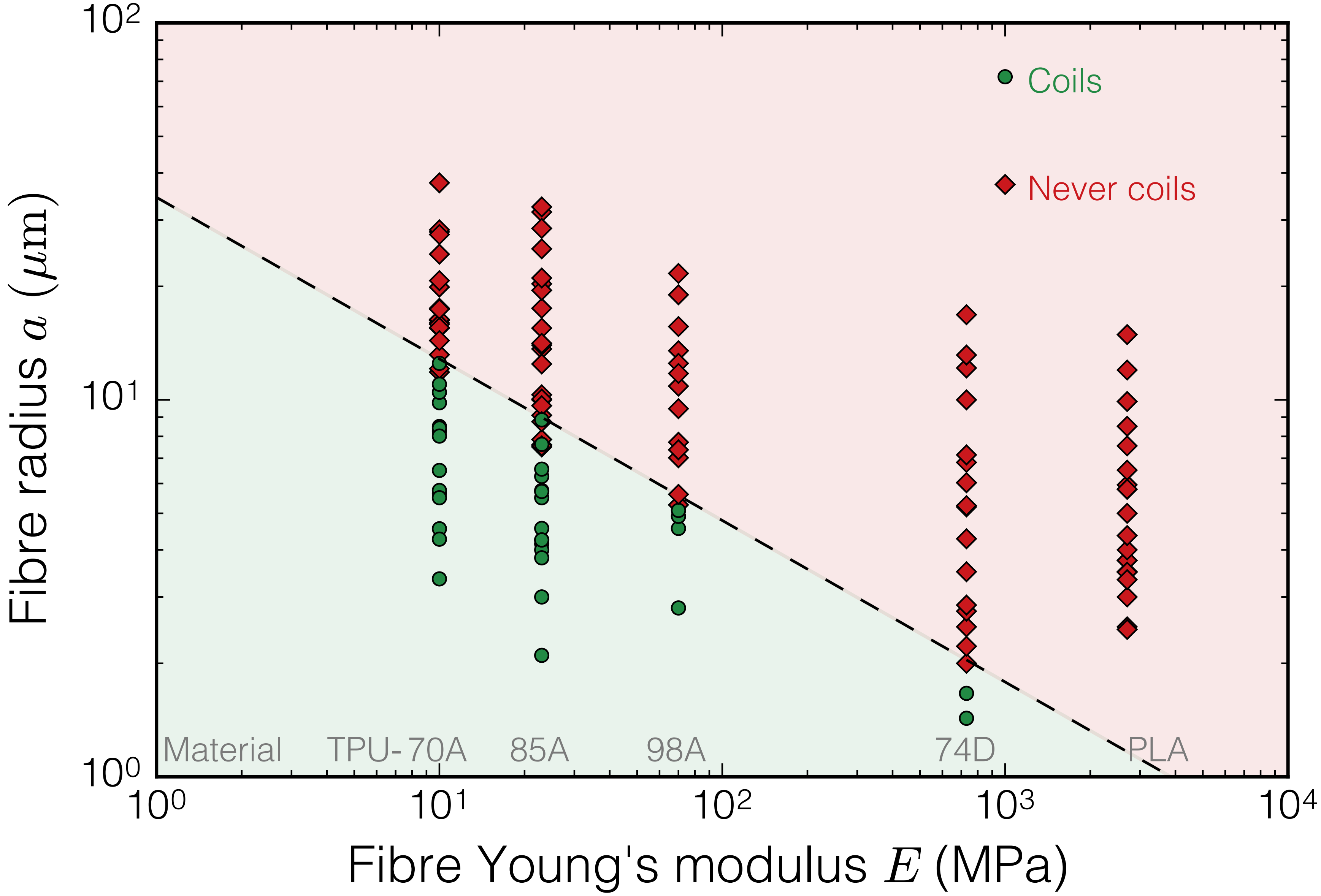
Fiber: Considered Young's modulus E = 70 MPa.

Droplet : γ=21 mN/m, ρg = 9600 N/m^3.

$a_{\max} = c(\rho g, \gamma) \cdot E^{-3/7}$

The coiling fibers

Phase diagram

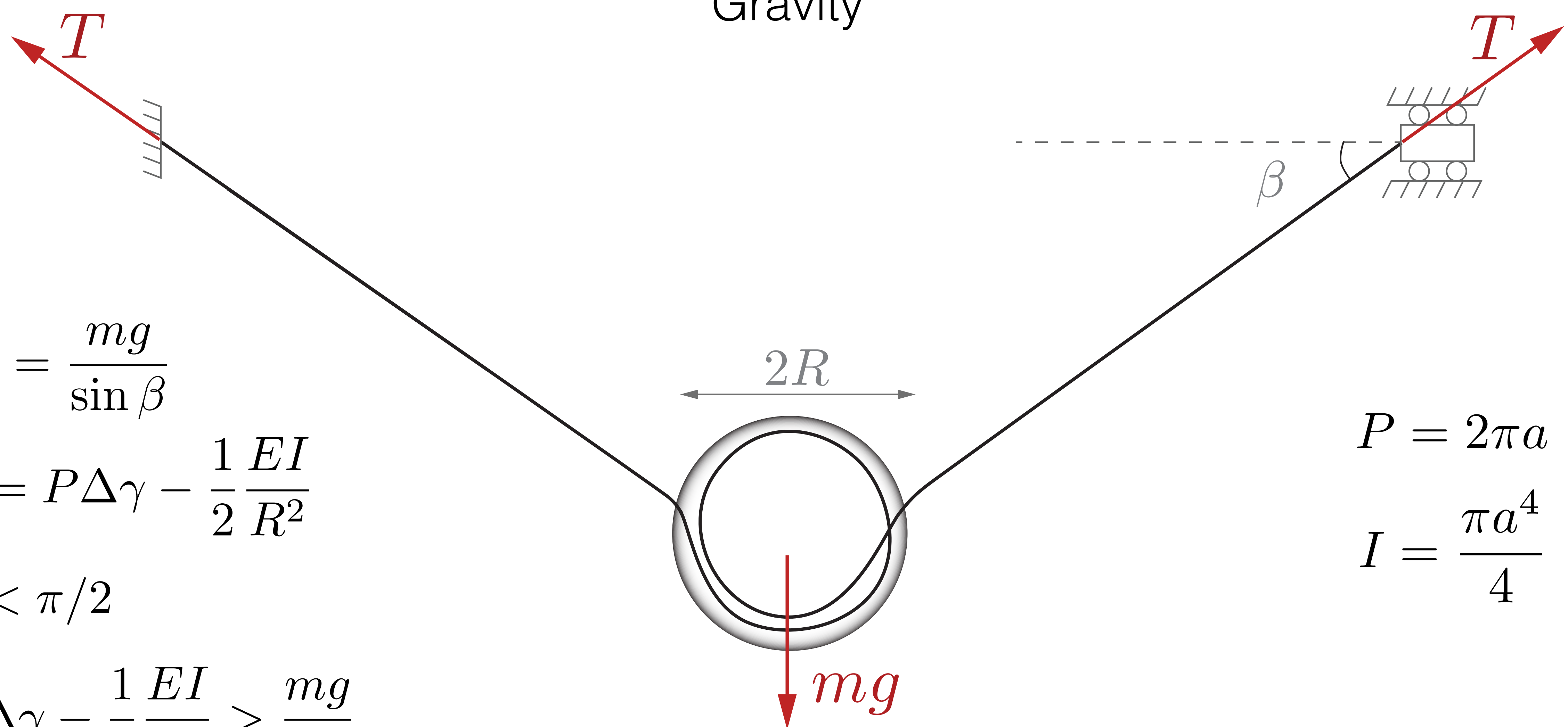


Phase diagram of **coilable** and **non-coilable** fibers.

Droplet : γ=21 mN/m, ρg = 9600 N/m^3.

Biggest coilable fibre

Gravity



$$2T = \frac{mg}{\sin\beta}$$

$$T = P\Delta\gamma - \frac{1}{2}\frac{EI}{R^2}$$

$$\beta < \pi/2$$

$$P\Delta\gamma - \frac{1}{2}\frac{EI}{R^2} > \frac{mg}{2}$$

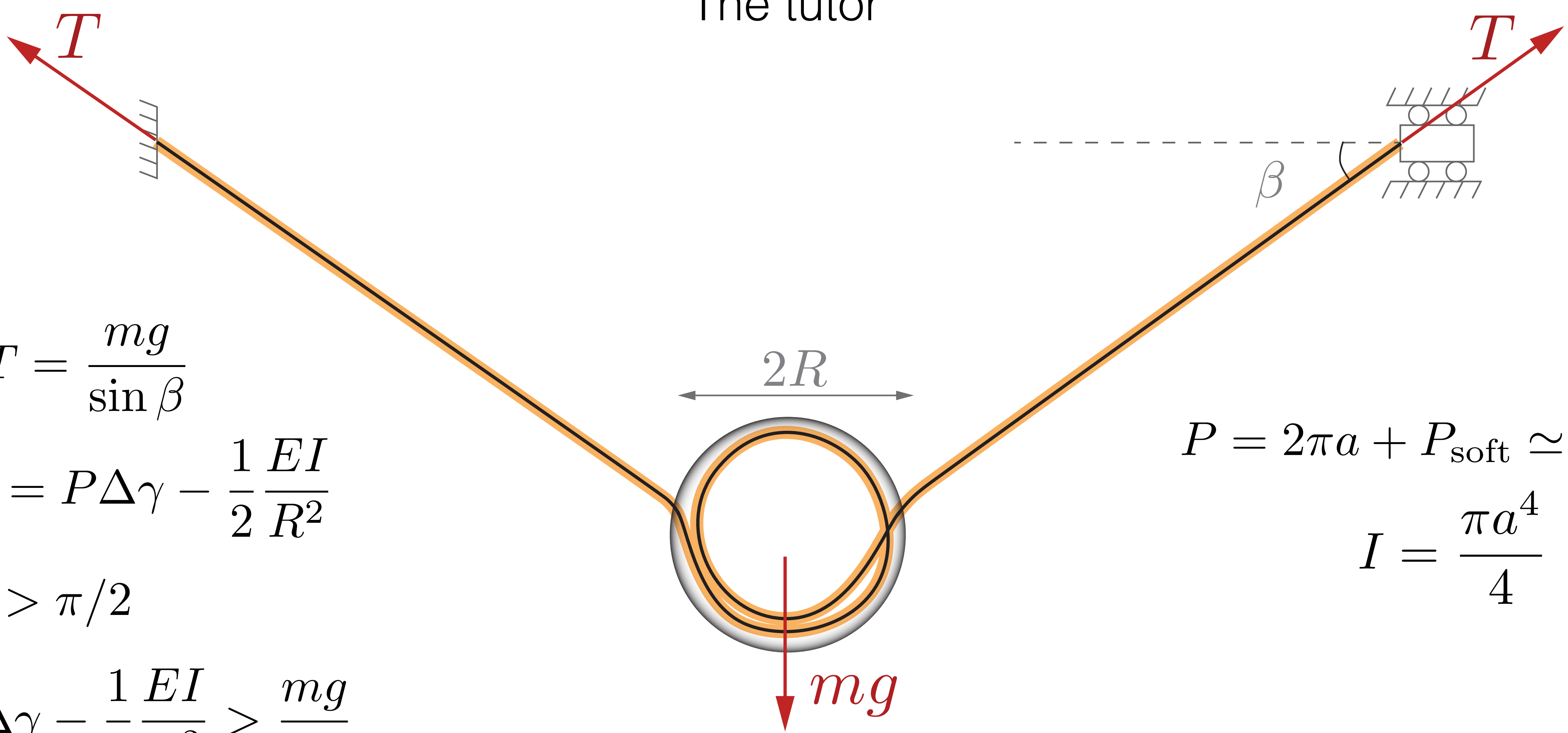
$$P = 2\pi a$$

$$I = \frac{\pi a^4}{4}$$

$$2\pi a\gamma - \frac{\pi a^4 E}{8R^2} > \frac{4}{3}\frac{\pi\rho g R^3}{2}$$

Biggest coilable fibre

The tutor



$$2T = \frac{mg}{\sin\beta}$$

$$T = P\Delta\gamma - \frac{1}{2}\frac{EI}{R^2}$$

$$\beta > \pi/2$$

$$P\Delta\gamma - \frac{1}{2}\frac{EI}{R^2} > \frac{mg}{2}$$

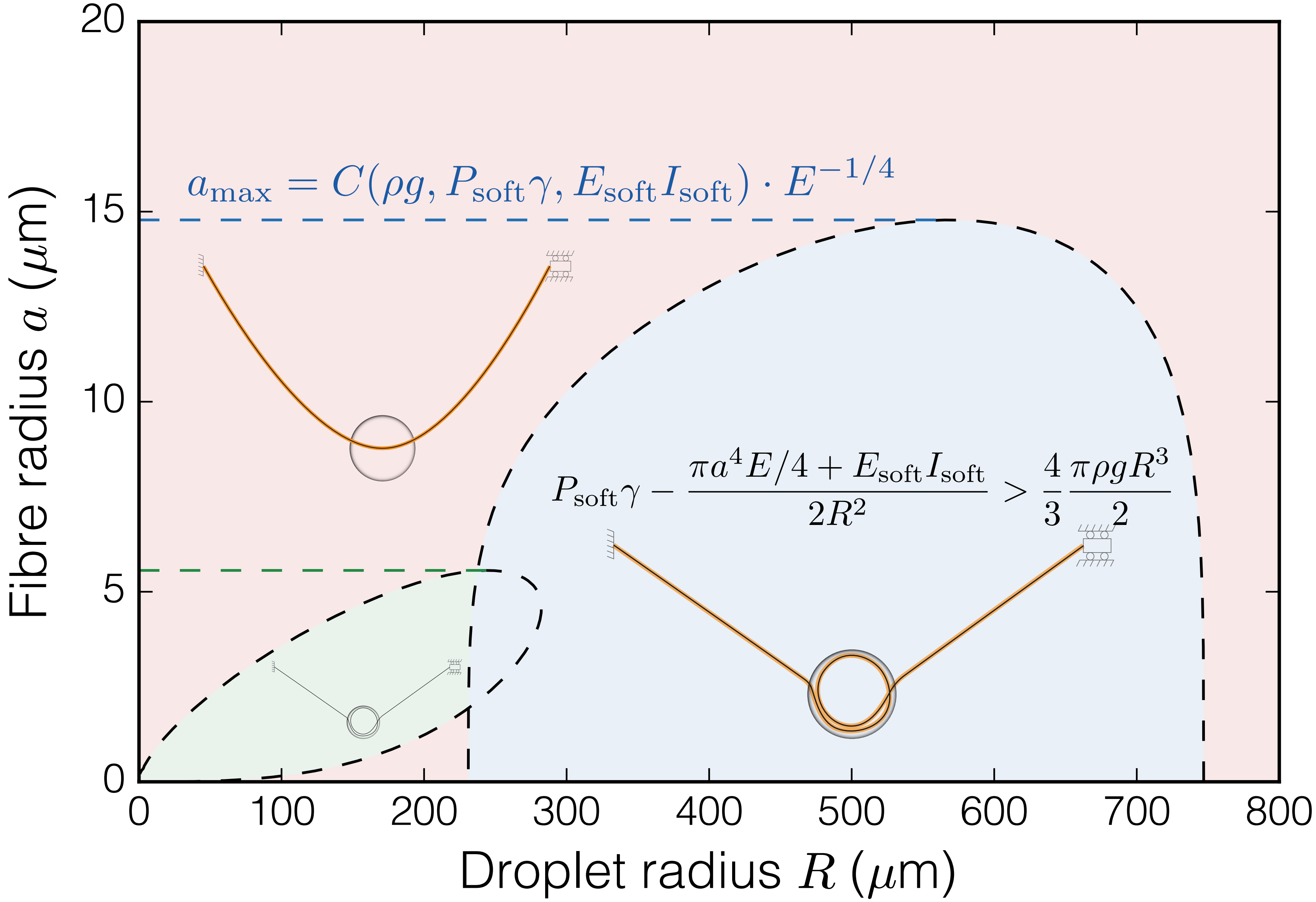
$$P = 2\pi a + P_{\text{soft}} \simeq P_{\text{soft}}$$

$$I = \frac{\pi a^4}{4}$$

$$P_{\text{soft}}\gamma - \frac{\pi a^4 E/4 + E_{\text{soft}} I_{\text{soft}}}{2R^2} > \frac{4}{3}\frac{\pi\rho g R^3}{2}$$

Biggest coilable fibre

The tutor



Phase diagram of **coiling** and **non-coiling** fibers with **gravity**.

Rigid fiber: Considered Young's modulus E = 70 MPa.

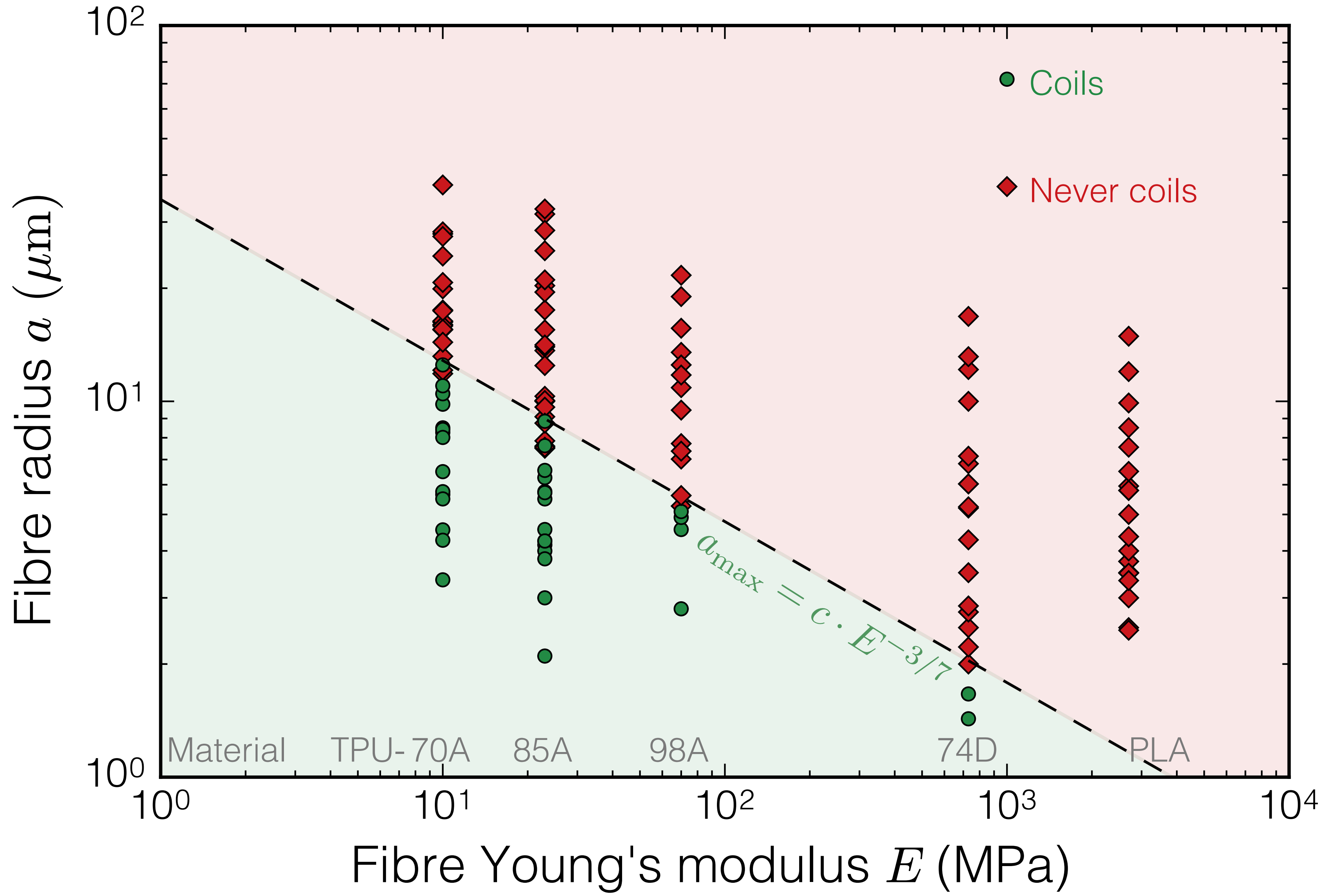
Soft fiber : Young's modulus E_{soft}=200 kPa, width w = 180 µm, height h = 50 µm.

Droplet : γ=21 mN/m, ρg = 9600 N/m^3.

$a_{\max} = C(\rho g, P_{\text{soft}}\gamma, E_{\text{soft}} I_{\text{soft}}) \cdot E^{-1/4}$

The coiling fibers

Phase diagram



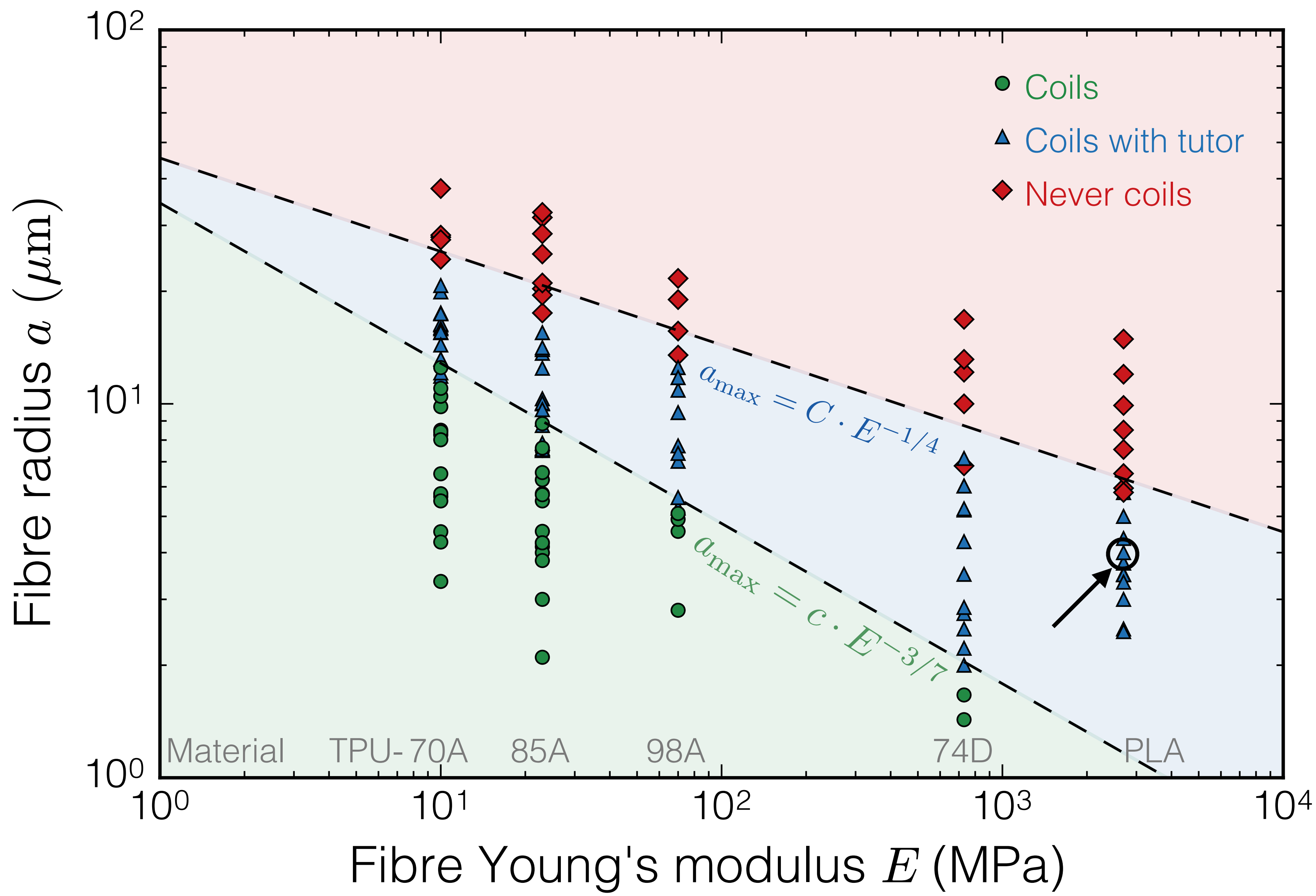
Phase diagram of **coilable** and **non-coilable** fibers.

Droplet : γ=21 mN/m.

$a_{\max} = C(\rho g, P_{\text{soft}}\gamma, E_{\text{soft}} I_{\text{soft}}) \cdot E^{-1/4}$

Coiling the uncoilable

The tutor



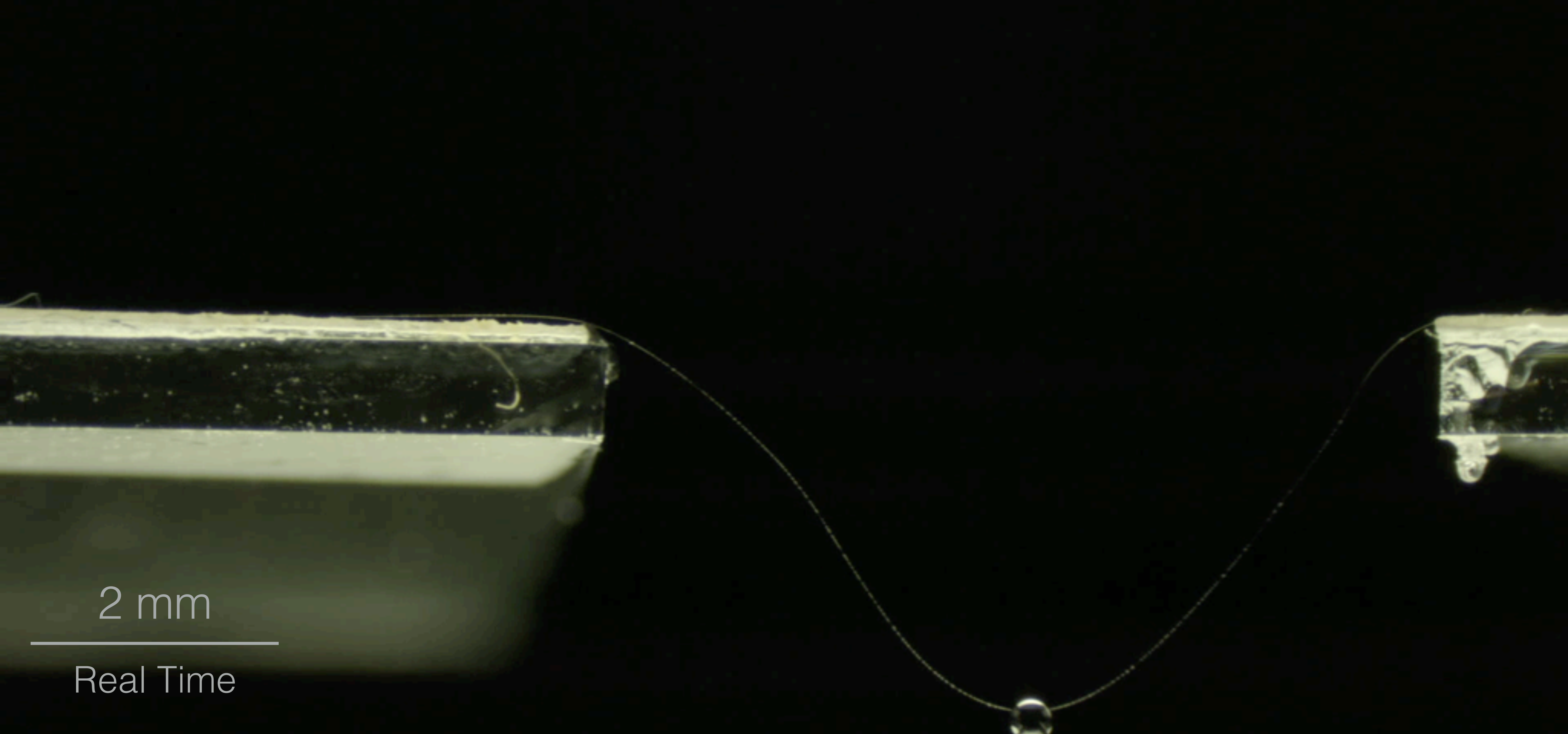
Phase diagram of **coilable** and **non-coilable** fibers with **tutor**.

Soft fiber : Young's modulus E_{soft}=200 kPa, width w = 180µm, height h = 50µm.

Droplet : γ=21 mN/m, ρg = 9600 N/m^3.

Coiling the uncoilable

Uncoilable



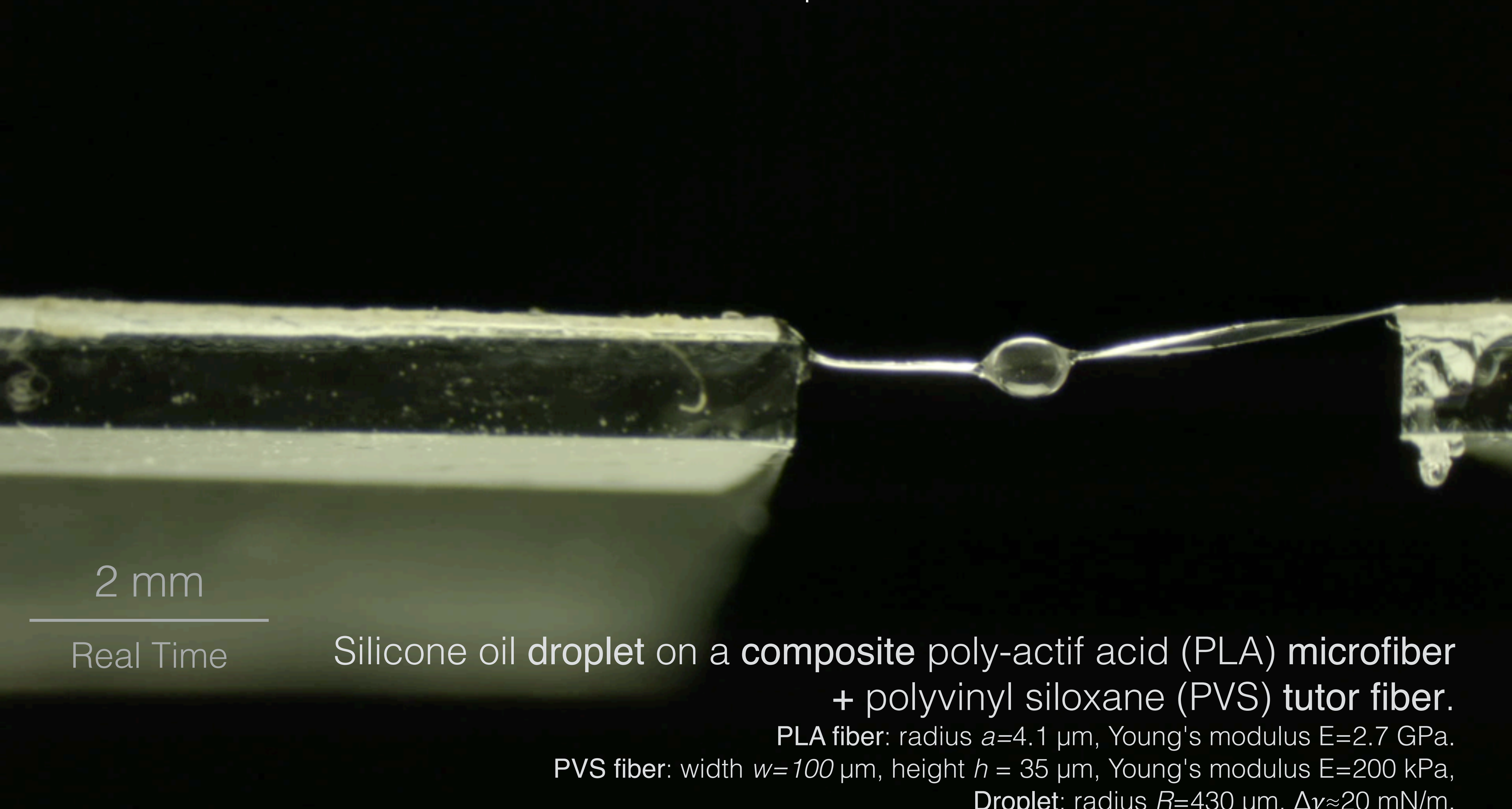
Silicone oil **droplet** on a poly-actif acid (PLA) **microfiber**.

Fiber: radius a=4.1 μm, Young's modulus E=2.7 GPa.

Droplet : radius R=256 μm, $\Delta\gamma\approx$20 mN/m.

Coiling the uncoilable

The coilabe composite fiber



Silicone oil **droplet** on a **composite** poly-actif acid (PLA) **microfiber**
+ polyvinyl siloxane (PVS) **tutor fiber**.

PLA fiber: radius a=4.1 μm, Young's modulus E=2.7 GPa.
PVS fiber: width w=100 μm, height h = 35 μm, Young's modulus E=200 kPa,
Droplet: radius R=430 μm, $\Delta\gamma\approx$20 mN/m.

Coiling the uncoilable

Coiled!

Thank you!